

# Sag-tension Calculations

A Tutorial Developed for the IEEE  
TP&C Line Design Subcommittee

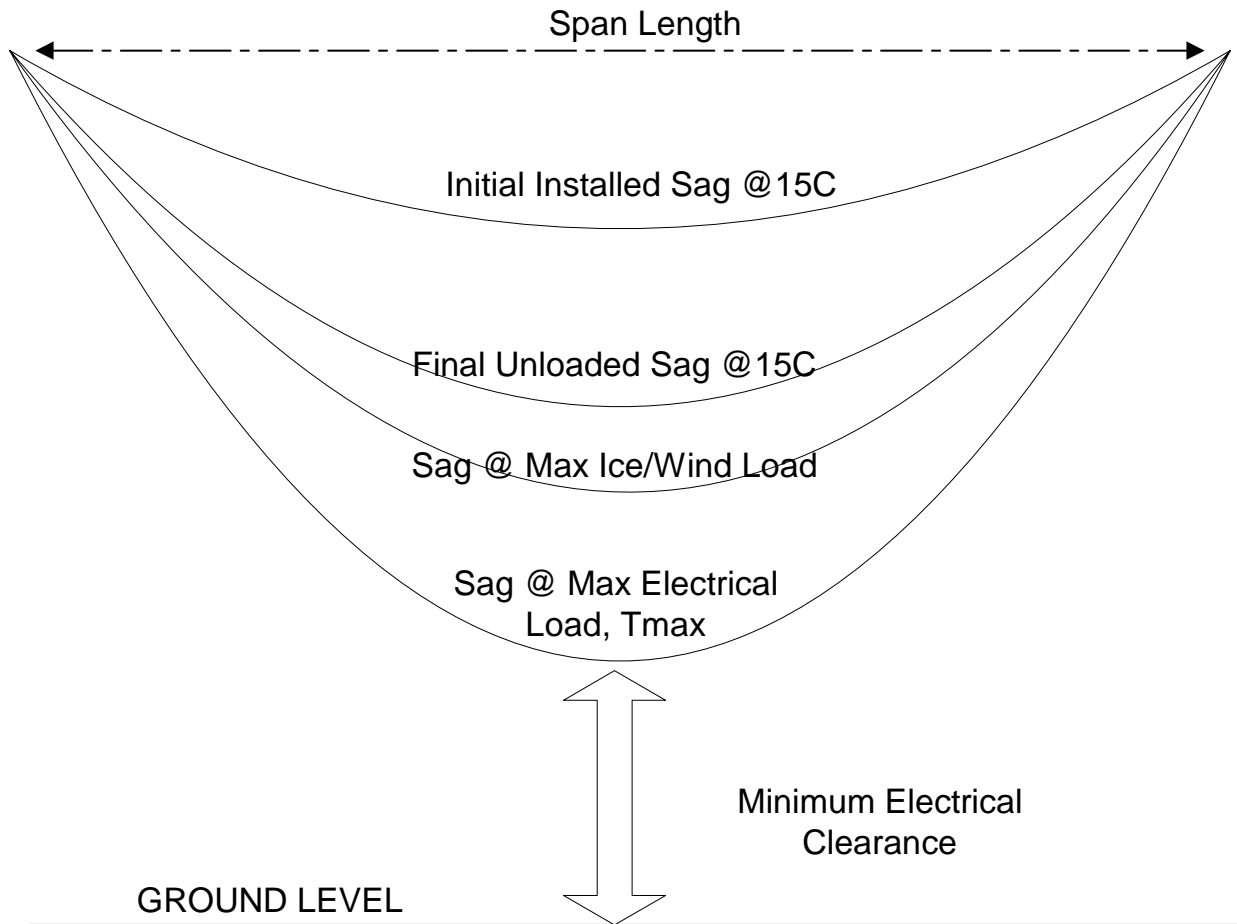
Based on a CIGRE WG B2.12 Technical  
Brochure under Development

Dale Douglass June, 2005

# CIGRE & IEEE Websites

- CIGRE WG B2.12 – Electrical Effects in Lines  
[http://www.geocities.com/wg\\_12/index.htm](http://www.geocities.com/wg_12/index.htm)
  - Technical Brochure 244 – Conductors for Uprating of Existing Lines
  - Probabilistic Ratings & Joints
- IEEE Towers Poles & Conductors  
[http://www.geocities.com/ieee\\_tpc/index.htm](http://www.geocities.com/ieee_tpc/index.htm)
  - IEEE Standard 738 – 1993
  - Panel Sessions Jan 28 (Las Vegas) June 4 (SF)

# Sag-tension Envelope



# SAG10 Calculation Table

| Design Points |      |      |      |        | Final |         |       | Initial |         |      |
|---------------|------|------|------|--------|-------|---------|-------|---------|---------|------|
| Temp          | Ice  | Wind | K    | Weight | Sag   | Tension | RTS   | Sag     | Tension | RTS  |
| F             | in   | psf  | lb/f | lb/f   | ft    | lb      | %     | ft      | lb      | %    |
| 0.            | .50  | 4.00 | .30  | 2.509  | 23.91 | 13142.  | 41.7  | 23.03   | 13647.  | 43.3 |
|               |      |      |      |        |       | 7064.A  |       |         | 7718.A  |      |
|               |      |      |      |        |       | 6078.S  |       |         | 5929.S  |      |
| 32.           | 1.00 | .00  | .00  | 3.715  | 29.08 | 16026.  | 50.9  | 29.08   | 16026.  | 50.9 |
|               |      |      |      |        |       | 8543.A  |       |         | 8543.A  |      |
|               |      |      |      |        |       | 7483.S  |       |         | 7483.S  |      |
| -20.          | .00  | .00  | .00  | 1.094  | 16.04 | 8532.   | 27.1  | 13.55   | 10103.  | 32.1 |
|               |      |      |      |        |       | 4361.A  |       |         | 6269.A  |      |
|               |      |      |      |        |       | 4171.S  |       |         | 3834.S  |      |
| 0.            | .00  | .00  | .00  | 1.094  | 17.39 | 7875.   | 25.0* | 14.43   | 9485.   | 30.1 |
|               |      |      |      |        |       | 3727.A  |       |         | 5820.A  |      |
|               |      |      |      |        |       | 4148.S  |       |         | 3665.S  |      |
| 60.           | .00  | .00  | .00  | 1.094  | 21.48 | 6379.   | 20.3  | 17.45   | 7847.   | 24.9 |
|               |      |      |      |        |       | 2125.A  |       |         | 4532.A  |      |
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|               |      |      |      |        |       | -94.A   |       |         | 610.A   |      |
|               |      |      |      |        |       | 4325.S  |       |         | 3827.S  |      |

\* Design Condition  
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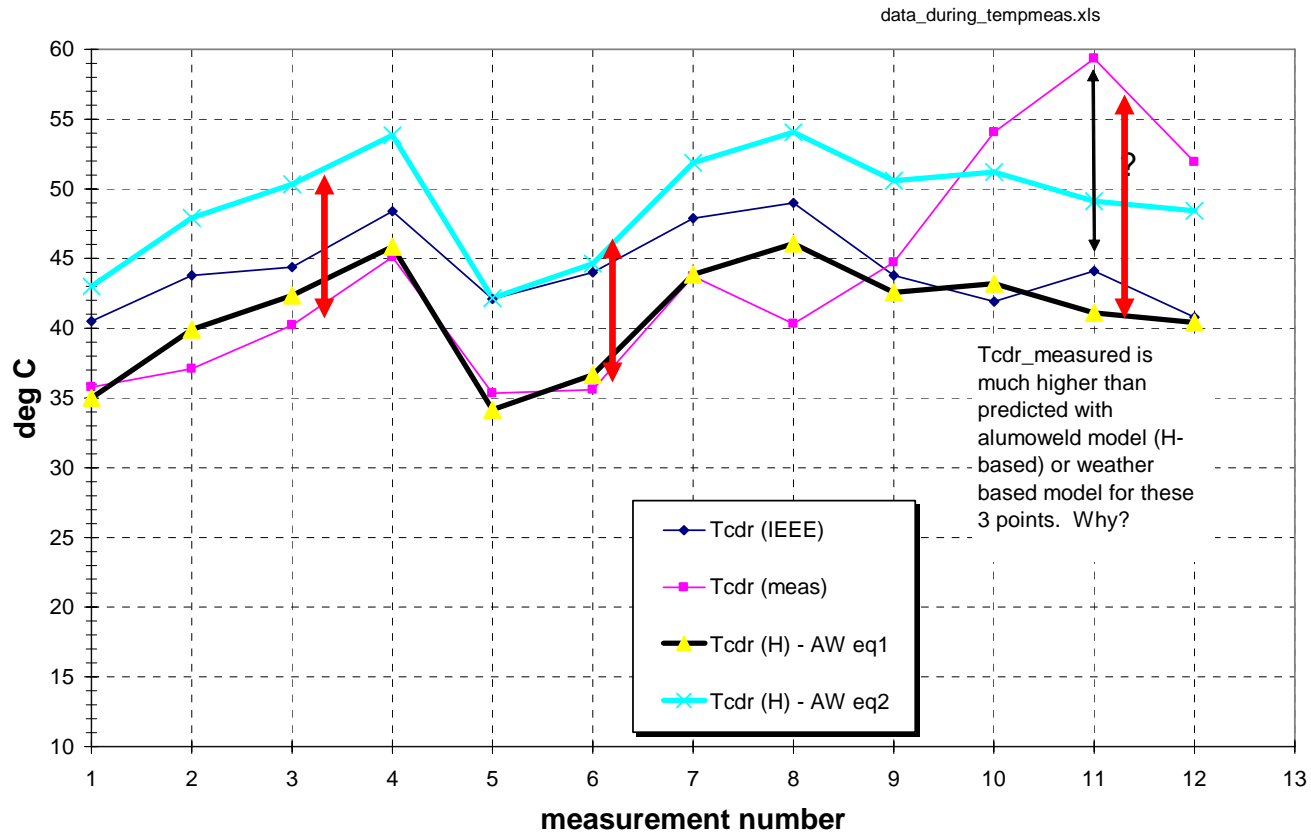
From Alcoa-Fujikura SAG10 program

# A Bit of Perspective



10C-15C Uncertainty

IPC measurements, 1997



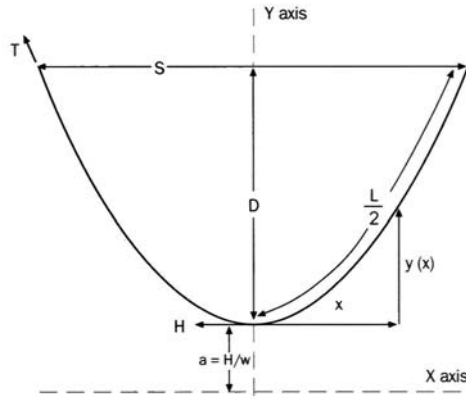
# Some Questions

- Why can we do calculations for a single span and use for an entire line section?
- How are initial and final conditions defined?
- Why not run the maximum tension to 60% as the NESC Code allows?
- Why do I see negative tensions (compression) in aluminum at high temperature?

# The Catenary Curve

- Hyperbolic Functions & Parabolas
- Sag vs weight & tension
- Length between supports
- What is Slack?
- What if the span isn't level?

# The Catenary – Level Span



$$y(x) = \frac{H}{w} \cdot \left[ \cosh\left(\frac{w \cdot x}{H}\right) - 1 \right] \cong \frac{w \cdot x^2}{2 \cdot H}$$

$$D = \frac{H}{w} \cdot \left\{ \cosh\left(\frac{w \cdot S}{2 \cdot H}\right) - 1 \right\} \cong \frac{w \cdot S^2}{8 \cdot H}$$

$$L = \left(\frac{2H}{w}\right) \sinh\left(\frac{Sw}{2H}\right) \cong S \left(1 + \frac{S^2 w^2}{24 H^2}\right)$$

# Catenary Sample Calcs for Drake ACSR

- 1.094 lbs/ft Bare Weight
- 31,500 lbs Rated Breaking Strength
- 600 ft span

$$D = \frac{6300}{1.094} \left[ \cosh \left( \frac{1.094 \cdot 600}{2 \cdot 6300} \right) \right] = 7.831 \text{ ft } (2.387 \text{ m})$$

$$L = \frac{2 * 6300}{1.094} \sinh \left( \frac{1.094 * 600}{2 * 6300} \right) = 600.272 \text{ ft } (182.963 \text{ m})$$

# Catenary Calculations

What Happens when the weight of  
the conductor changes

# Ice & Wind Loading

- Radial ice (Quebec)
- Wind Pressure (Florida)
- Wind & Ice Combined (Illinois)

# What about changes in loading?



| <i>NESC Loading District</i>   |              |               |              |                             |
|--|--------------|---------------|--------------|-----------------------------|
|  | <i>Heavy</i> | <i>Medium</i> | <i>Light</i> | <i>Extreme wind loading</i> |
| Radial thickness of ice<br>(in)<br>(mm)                                    | 0.50<br>12.5 | 0.25<br>6.5   | 0<br>0       | 0<br>0                      |
| Horizontal wind pressure<br>(lb/ft <sup>2</sup> )<br>(Pa)                  | 4<br>190     | 4<br>190      | 9<br>430     | See Fig 2-4                 |
| Temperature<br>(°F)<br>(°C)  | 0<br>-20     | +15<br>-10    | +30<br>-1    | +60<br>+15                  |
| NESC safety factors<br>to be added to<br>the resultant<br>(lb/ft)<br>(N/m) | 0.30<br>4.40 | 0.20<br>2.50  | 0.05<br>0.70 | 0.0<br>0.0                  |

# Iced Conductor Weight

$$W_{ice} = 1.244t ( D_c + t )$$

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| ACSR<br>Conductor            | $D_c$ ,<br>in | $W_{bare}$ ,<br>lb/ft | $W_{ice}$ ,<br>lb/ft | $\frac{W_{bare} + W_{ice}}{W_{bare}}$ |
|------------------------------|---------------|-----------------------|----------------------|---------------------------------------|
| #1/0 AWG -6/1<br>"Raven"     | 0.398         | 0.1452                | 0.559                | 4.8                                   |
| 477 kcmil-26/7<br>"Hawk"     | 0.858         | 0.6570                | 0.845                | 2.3                                   |
| 1590 kcmil-54/19<br>"Falcon" | 1.545         | 2.044                 | 1.272                | 1.6                                   |

# What happens when the conductor weight changes?

- Bare weight of Drake ACSR is 1.094 lb/ft
- Iced weight is:
  - $1.094 + 1.244 * 1.0 * (1.108 + 1.0) = 3.60$  lb/ft
- Tension increases by a factor of 3.6 unless the length of the conductor changes.

# SAG10 Calculation Table

| Design Points |      |      |      | Final  |       |         |       | Initial |         |      |
|---------------|------|------|------|--------|-------|---------|-------|---------|---------|------|
| Temp          | Ice  | Wind | K    | Weight | Sag   | Tension | RTS   | Sag     | Tension | RTS  |
| F             | in   | psf  | lb/f | lb/f   | ft    | lb      | %     | ft      | lb      | %    |
| 0.            | .50  | 4.00 | .30  | 2.509  | 23.91 | 13142.  | 41.7  | 23.03   | 13647.  | 43.3 |
|               |      |      |      |        |       | 7064.A  |       |         | 7718.A  |      |
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| 32.           | 1.00 | .00  | .00  | 3.715  | 29.08 | 16026.  | 50.9  | 29.08   | 16026.  | 50.9 |
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|               |      |      |      |        |       | 4171.S  |       |         | 3834.S  |      |
| 0.            | .00  | .00  | .00  | 1.094  | 17.39 | 7875.   | 25.0* | 14.43   | 9485.   | 30.1 |
|               |      |      |      |        |       | 3727.A  |       |         | 5828.A  |      |
|               |      |      |      |        |       | 4148.S  |       |         | 3665.S  |      |
| 60.           | .00  | .00  | .00  | 1.094  | 21.48 | 6379.   | 20.3  | 17.45   | 7847.   | 24.9 |
|               |      |      |      |        |       | 2125.A  |       |         | 4532.A  |      |
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|               |      |      |      |        |       | -94.A   |       |         | 610.A   |      |
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\* Design Condition  
June 6/13/05

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From Alcoa-Fujikura SAG10 program

# Conductor tension limits

- Avoiding tension failure (Safety factor)
- Limiting vibration (H/w, %RBS)
- Designing with less sag

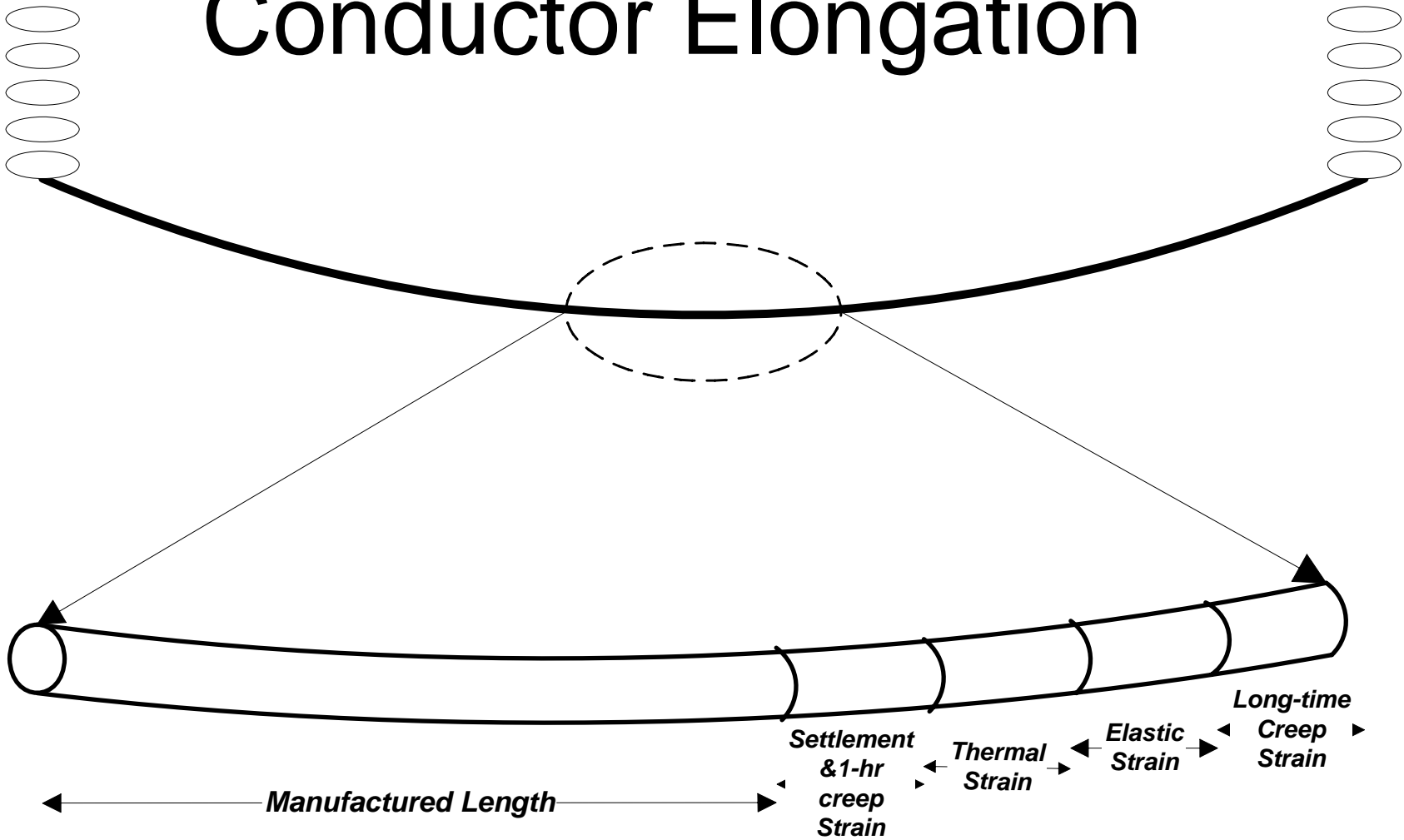
# Tension Limits and Sag

| Tension at 15C unloaded initial - %RTS | Tension at max ice and wind load - %RTS | Tension at max ice and wind load - kN | Initial Sag at 100C - meters | Final Sag at 100C - meters |
|--|---|---------------------------------------|------------------------------|----------------------------|
| 10                                     | 22.6                                    | 31.6                                  | 14.6                         | 14.6                       |
| 15                                     | 31.7                                    | 44.4                                  | 10.9                         | 11.0                       |
| 20                                     | 38.4                                    | 53.8                                  | 9.0                          | 9.4                        |
| 25                                     | 43.5                                    | 61.0                                  | 7.8                          | 8.4                        |

# Conductor Elongation

- Elastic elongation (springs)
- Settlement & Short-term creep (before sagging)
- Thermal elongation
- Long term creep (After sagging, over the life of the line)

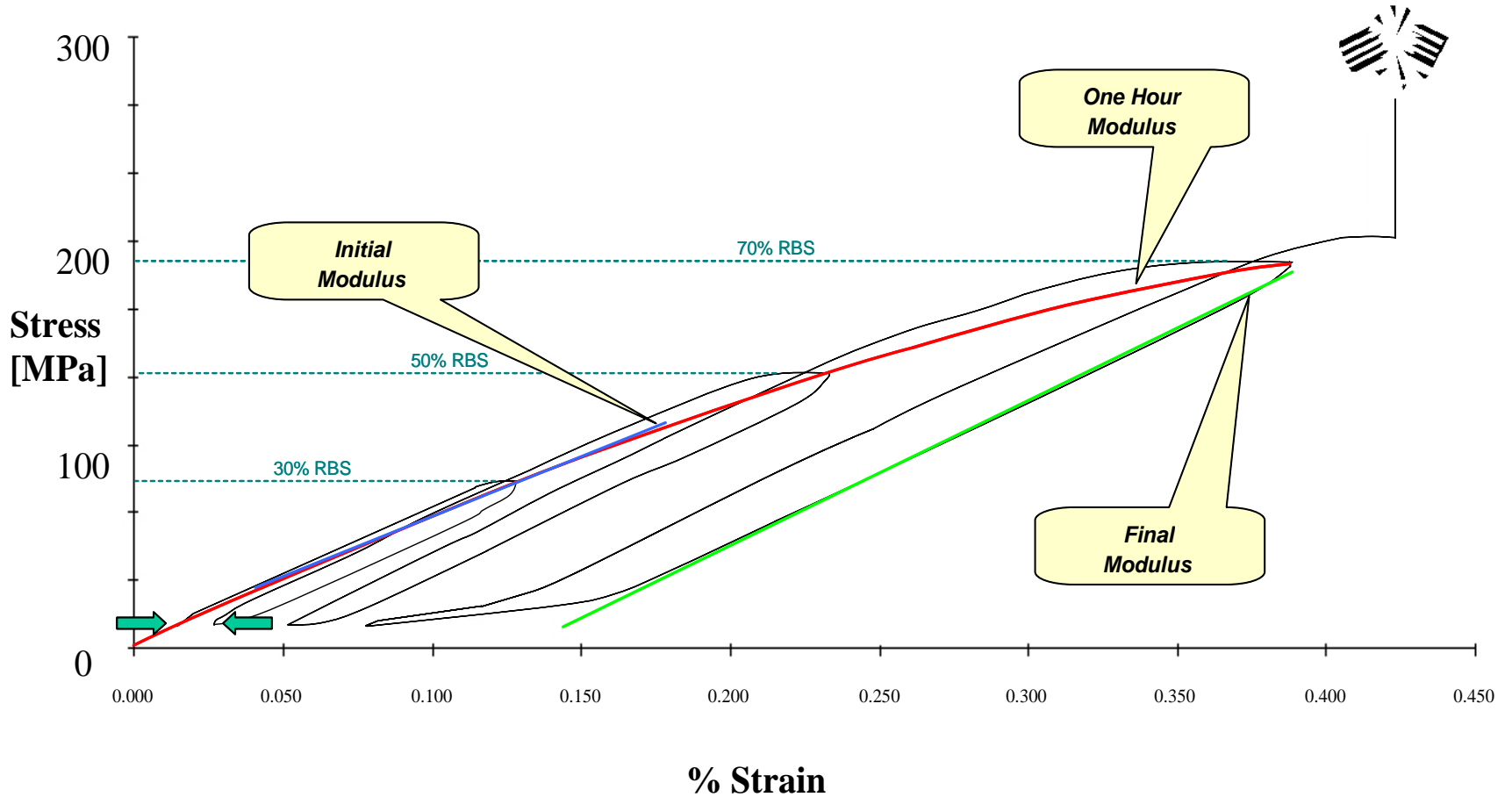
# Conductor Elongation



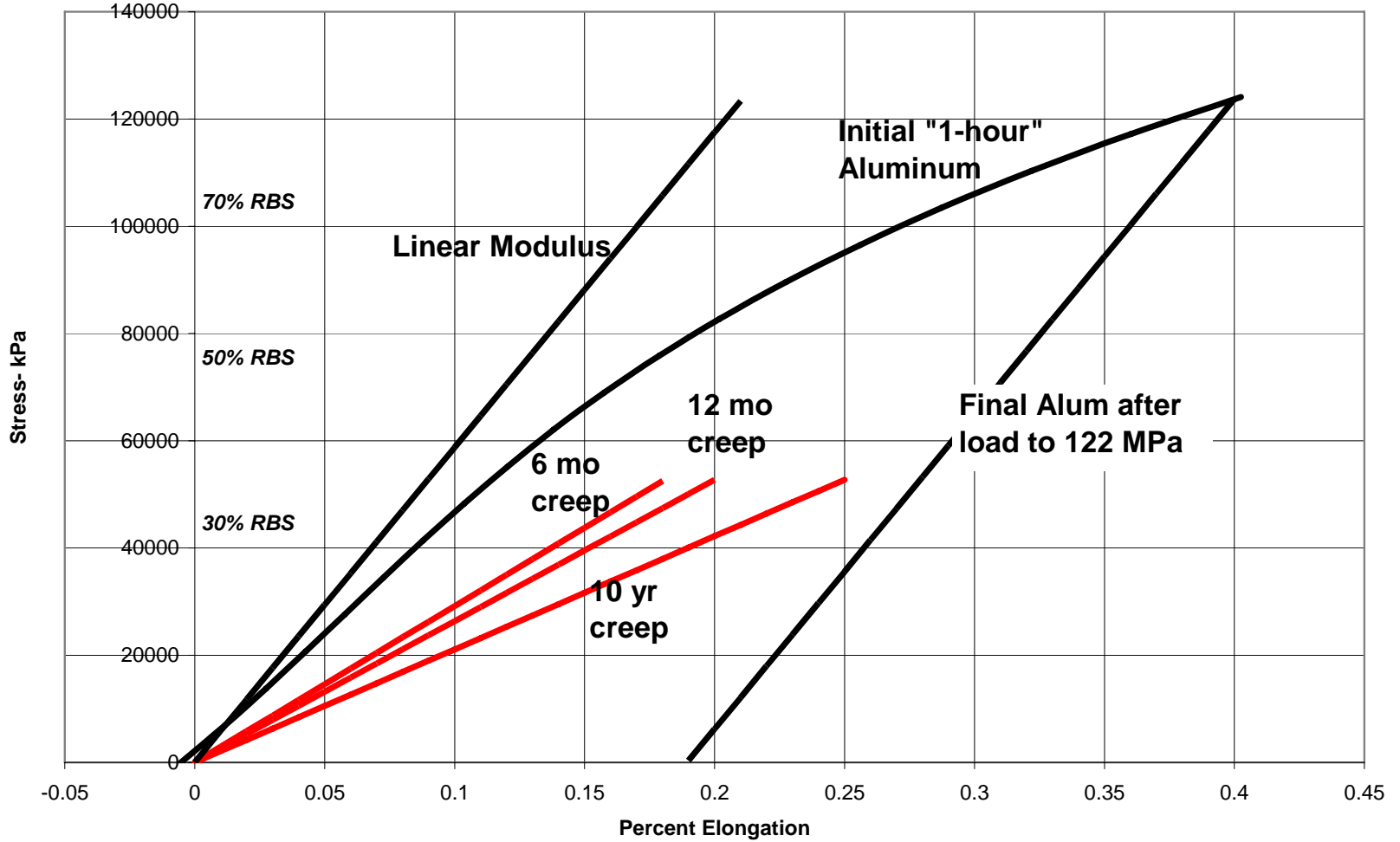
# Thermal Elongation

|  | <b>International<br/>Annealed Copper<br/>Standard</b> | <b>Commercial<br/>Hard-Drawn<br/>Copper Wire</b> | <b>Standard<br/>1350-H19<br/>Aluminum<br/>Wire</b> | <b>Galv.<br/>Steel Core<br/>Wire</b> |
|--|---|--|--|--------------------------------------|
| Conductivity,<br>% IACS @ 20°C                         | 100.00  | 97.00  | 61.2   | 8.0                                  |
| Coefficient of<br>Linear Expansion<br>$10^{-6}$ per °F | 9.4   | 9.4  | 12.8   | 6.4                                  |

# Stress-Strain Test

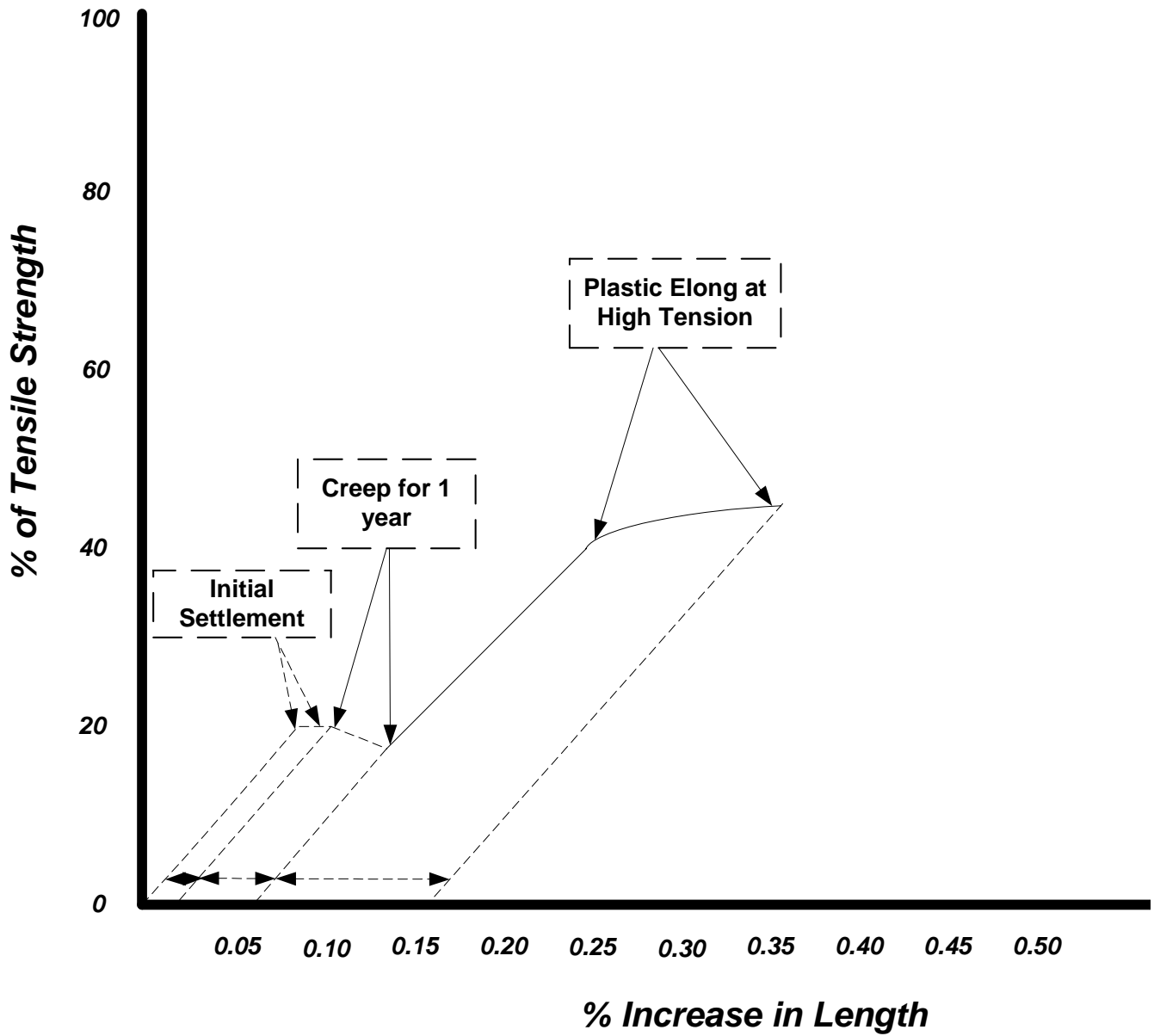


Stress-strain & creep elongation curves for 37 strand A1 conductor



# Conductor Elongation

- Elastic elongation (reversible)
- Settlement & Short-term creep (permanent)
- Thermal elongation (reversible)
- Long term creep (permanent after years or high loads)



# SAG10 Calculation Table

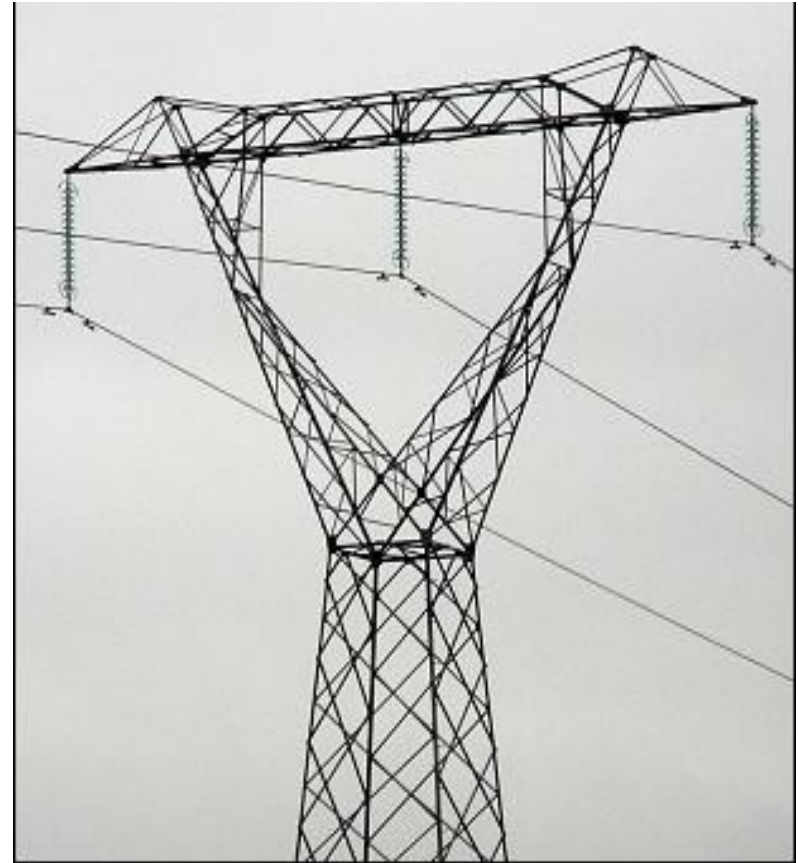
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|---------------|------|------|------|--------|-------|---------|-------|---------|---------|------|
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\* Design Condition  
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From Alcoa-Fujikura SAG10 program

# What is a ruling span?

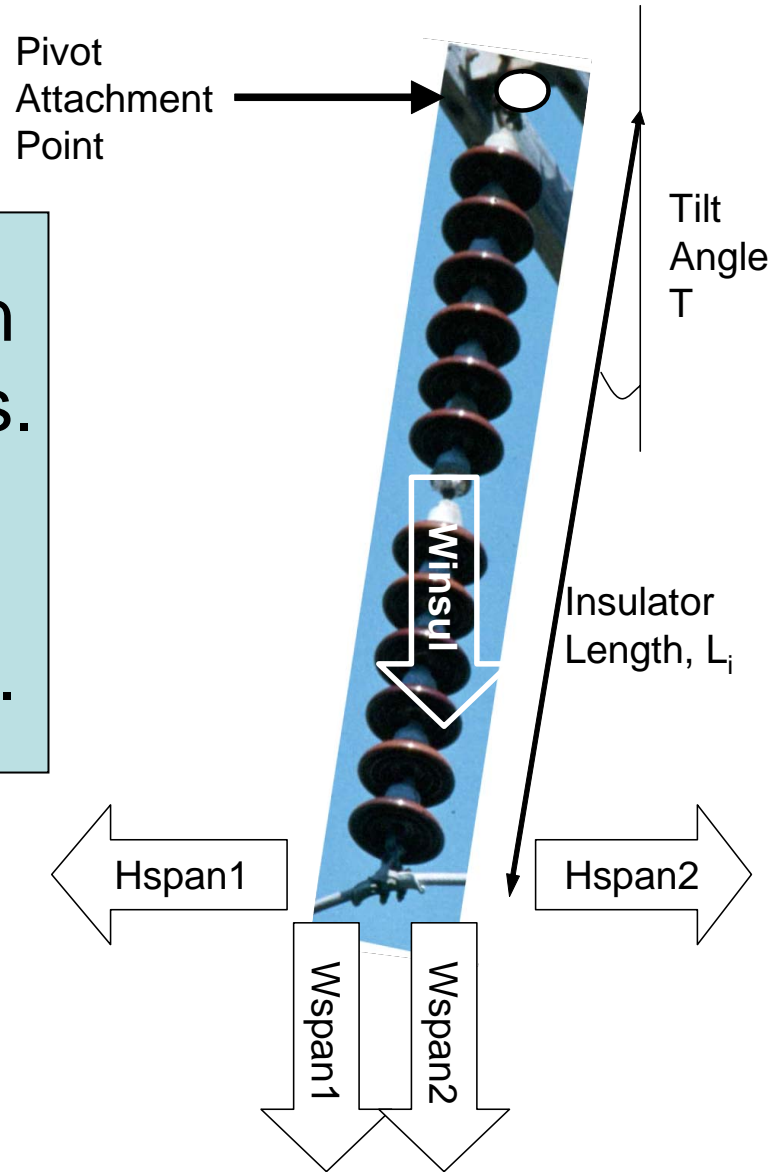


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Tension equalization  
at suspension points.

The basis of the  
ruling span concept.



# The “Ruling Span”

$$RS = \sqrt{\frac{S_1^3 + S_2^3 + \dots + S_n^3}{S_1 + S_2 + \dots + S_n}}$$

$$RS = \sqrt{\frac{600^3 + 900^3 + 600^3}{600 + 900 + 600}} = 745 \text{ ft}$$

- Based on Tension equalization
- Used for Stringing sags
  - Sag =  $(w/8H) * S^2$

# Sag-tension Calculations - Deliverables

- **Maximum sag** so that clearance to ground and other conductors can be maintained.
- **Maximum tension** so that structures can be designed to withstand it.
- **Minimum sag** to control structure uplift problems.
- **H/w during “coldest month”** to limit aeolian vibration.

# Summary of Some Key Points

- Tension equalization between suspension spans allows use of the ruling span
- Initial and final conditions occur at sagging and after high loads and multiple years
- For large conductors, max tension is typically below 60% in order to limit wind vibration & uplift
- Negative tensions (compression) in aluminum occur at high temperature for ACSR because of the 2:1 diff in thermal elongation between alum & steel

# General Sag-Ten References

- Aluminum Association Aluminum Electrical Conductor Handbook Publication No. ECH-56"
- Southwire Company "Overhead Conductor Manual"
- Barrett, JS, Dutta S., and Nigol, O., *A New Computer Model of A1/S1A (ACSR) Conductors*, IEEE Trans., Vol. PAS-102, No. 3, March 1983, pp 614-621.
- Varney T., Aluminum Company of America, "Graphic Method for Sag Tension Calculations for A1/S1A (ACSR) and Other Conductors.", Pittsburg, 1927
- Winkelman, P.F., "Sag-Tension Computations and Field Measurements of Bonneville Power Administration, AIEE Paper 59-900, June 1959.
- IEEE Working Group, "Limitations of the Ruling Span Method for Overhead Line Conductors at High Operating Temperatures". Report of IEEE WG on Thermal Aspects of Conductors, IEEE WPM 1998, Tampa, FL, Feb. 3, 1998
- Thayer, E.S., "Computing tensions in transmission lines", Electrical World, Vol.84, no.2, July 12, 1924
- Aluminum Association, "Stress-Strain-Creep Curves for Aluminum Overhead Electrical Conductors," Published 7/15/74.
- Barrett, JS, and Nigol, O., *Characteristics of A1/S1A (ACSR) Conductors as High Temperatures and Stresses*, IEEE Trans., Vol. PAS-100, No. 2, February 1981, pp 485-493
- Electrical Technical Committee of the Aluminum Association, "A Method of Stress-Strain Testing of Aluminum Conductor and ACSR" and "A Test Method for Determining the Long Time Tensile Creep of Aluminum Conductors in Overhead Lines", January, 1999, The aluminum Association, Washington, DC 20006, USA.
- Harvey, JR and Larson RE. *Use of Elevated Temperature Creep Data in Sag-Tension Calculations*. IEEE Trans., Vol. PAS-89, No. 3, pp. 380-386, March 1970
- Rawlins, C.B., "Some Effects of Mill Practice on the Stress-Strain Behaviour of ACSR", IEEE WPM 1998, Tampa, FL, Feb. 1998.

# The End

## A Sag-tension Tutorial

Prepared for the IEEE TP&C  
Subcommittee by Dale Douglass